

Modeling and Numerical Investigation of the Biomechanical Interaction for Human-Rifle System

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This paper represents the description of a complex mathematical model of biomechanical interaction for human-rifle system during shooting. The model is developed by finite element method using bar elements. And three typical shooting positions, i.e. standing, kneeling and prone are used. Characteristics of interior/exterior ballistics and behaviors of human-rifle system are evaluated by this model, which takes into account the influence of environment, bullet, powder, barrel geometry parameters and anthropological parameters. The results of this study can be applied to anthropology, biomechanics, medical science, gait analysis, interior ballistics and exterior ballistics.

Key Words : Human-Rifle System, Small Arms, Standing Position, Kneeling Position, Prone Position, Ballistics, Dispersion Characteristics

1. Introduction

In the small caliber weapon system, to determine the dispersion of bullets is one of the most important characteristics of firing accuracy. Generally an improvement of firing accuracy depends upon not only armament characteristics but also individual biological, anthropological and physical human characteristics. Experimental tests to evaluate the characteristics of weapon are spent too much time, cost and manpower. Especially it is difficult to collect and analyze the data. Therefore the study of biomechanical characteristics using human-rifle modeling and numerical investigation is needed.

It is known that the major reasons for additional dispersions of bullet on target are the vibrations (displacements) of the human body-rifle system and foremost hands, breast and another parts of the body. Vertical and horizontal vibrations exist because of the elastic connection between the rifle and parts of the human body during firing in muscle-skeleton elements with the elastic-plastic deformations. These vibrations are so lightly damped that the human hand with rifle and his body cannot return to their initial positions. As a result, the aiming angles from shot to shot will change during burst firing. In the case of high fire-rate shooting due to small time interval between shots, correcting of the aiming angle is impossible due to biological and psycho-motor reasons. During shooting with small caliber armament, the displacements of human-rifle system leads to deviations of the initial aiming angle of vertical as well as horizontal directions and influence the jump angle of the bullet. Also this angle has a probability depending on random factors. Therefore displacements are required to

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solve in joint settlement tasks of interior ballistics (for calculation of loads formed by rifle and transferring to places of rifle contact with human body) and dynamic of biomechanical interaction of the human body and rifle during single or burst shooting. Calculation of spatial linear and angular displacements appears in Support-Locomotor Apparatus of Human (SLAH) and rifle connected with them during impulse dynamical action and their influence on firing accuracy. The last criteria is to calculate parameters of exterior ballistics with taking into account the interaction of human-rifle biomechanical system.

2. Characteristics of Mathematical Model

2.1 Interior ballistics model

A mathematical model of interior ballistics is based on the lumped method approach (Zakharenkov, 2000). The procedure of modelling is powder burning, motion of projectile and gas flow. The effects of gas consumption through port-hole in barrel for provision of the work of gas automatic mechanism are taken into account. The system of interior ballistic equations is solved by the fourth order Runge-Kutta method. The solution can be found with the determined settlements or probability settlements. In the last case, the Monte Carlo method is used.

2.2 Biomechanical model of human-rifle system

The biomechanical model is based on the investigations (Arseniev et al., 1995) devoted study to the different aspects of biomechanic human body movement, which is kinematical and dynamical analysis of spatial movements of human body with a rifle under action of exterior and interior forces. The characteristics of human muscles are determined by the sensitivity analysis method.

2.3 Kinematics and dynamics

Kinematics is described as follows. The major equation of generalized displacements $\{\Delta\}$ of skeleton-muscle system depends on statical ap-

plication of forces $\{P\}$

$$[K] \cdot \{\Delta\} = \{P\} \tag{1}$$

where $[K]$ is a global matrix of system stiffness, $\{\Delta\}$ is solved generalized displacements vector and $\{P\}$ is generalized exterior forces vector. The elements of the global stiffness matrix $[K]$ are calculated by common rules of the finite element method (FEM) and represented as :

$$K = \sum_{e=1}^n \cdot \hat{k}_{ij}^e \tag{2}$$

where k_{ij}^e is elements of local stiffness matrix oriented in global system of coordinate, i and j are the numbers of generalized displacements in local system, and n is the number of finite elements.

The structural mechanical displacements of bar elements are represented in Fig. 1

Connection between local stiffness matrix of bar element and arbitrary oriented in global XYZ stiffness matrix of the same element in local xyz $[k^e]$ is carried out by way of guide cosine matrix $[L]$ in accordance with dependence. $[L]$ is matrix of direction cosines.

$$[\hat{k}^e] = [L]^T \cdot [k^e] \cdot [L] \tag{3}$$

where

$$[L] = \begin{bmatrix} [l] & & & \\ & [l] & & \\ & & [l] & \\ & & & [l] \end{bmatrix}, [l] = \begin{bmatrix} l_{xx} & l_{xy} & l_{xz} \\ l_{yx} & l_{yy} & l_{yz} \\ l_{zx} & l_{zy} & l_{zz} \end{bmatrix}$$

Each element of local stiffness matrix k_{ij}^e re-

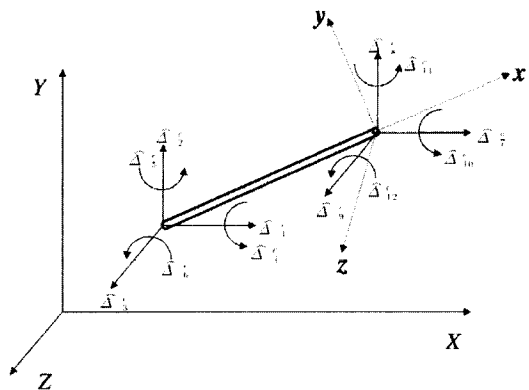


Fig. 1 The human muscle-skeleton elements modelled by bar element

presents a force (or reaction) appearing in i direction for unit displacement along j direction. Full stiffness matrix is symmetrical with the exception of the following elements, and the others are zero.

$$k_{1,1}^e = -k_{1,7}^e = k_{7,7}^e = -k_{7,1}^e = EF/L \quad (4.1)$$

$$k_{2,2}^e = -k_{2,8}^e = k_{8,8}^e = -k_{8,2}^e = 12EF/L^3 \quad (4.2)$$

$$k_{2,6}^e = k_{2,12}^e = k_{6,2}^e = k_{12,6}^e = -k_{6,8}^e = -k_{8,6}^e \\ = -k_{8,12}^e = k_{12,8}^e = 6EI_z/L^2 \quad (4.3)$$

$$k_{3,3}^e = k_{9,9}^e = -k_{9,3}^e = -k_{3,9}^e = 12EI_y/L^3 \quad (4.4)$$

$$k_{3,5}^e = k_{3,11}^e = -k_{5,11}^e = -k_{11,9}^e = k_{5,3}^e = -k_{5,11}^e \\ = -k_{11,9}^e = 6EI_y/L^2 \quad (4.5)$$

$$k_{4,4}^e = -k_{4,10}^e = -k_{10,4}^e = k_{10,10}^e = GI_x/L \quad (4.6)$$

$$k_{5,5}^e = k_{11,11}^e = 4EI_y/L \quad (4.7)$$

$$k_{6,6}^e = k_{12,12}^e = 4EI_z/L \quad (4.8)$$

$$k_{8,11}^e = k_{11,5}^e = 2EI_y/L \quad (4.9)$$

$$k_{6,12}^e = k_{12,6}^e = 2EI_z/L \quad (4.10)$$

where L is length of element, E and G are modula of elasticity and rigidity, F is area of cross-section, I_x , I_y and I_z are inertia moment of elements on cross-section area about main central axes.

To determine natural frequencies and modes of vibration, the human-rifle FEM (bar element system) is provided by modified solution of values $([K] - \omega^2[M]) \cdot \{U\}$, where $[M]$ is equal to $[M]_1 + [M]_2$, $[M]_1$ is the diagonal mass matrix of system taking into account mass and their moment of inertia, $[M]_2$ is the mass matrix of the system taking into account distributed mass of elements; ω_i is the spectrum of natural frequencies ($i=1, \dots, n$), and $\{U\}_i$ is the natural vector (normalized form of vibrations of the system on i -th frequency).

With the given equations, the geometrical characteristics of human-rifle system and current position can be determined. It is then impossible to compare kinematical characteristics of different riflemen on height, mass and others, and find optimal particularities of movement of rifleman. In order to describe the kinematical movements of the rifleman, the anatomical characteristics of joints and human apparatus are used. In addition,

to calculate the trajectory of the body working with the rifle during shooting for one or another pose, the angular limitations connected with available mobility of joints are used. The mobility of joints depends on personal skeleton-muscle constitution such as short/thick, middle/normal, tall/thin and so forth.

It is also necessary to take into account mathematical model dynamics of spatial human-rifle movement, supposing the use of interior or exterior dynamical force characteristics. The exterior forces which act on human-rifle mechanism is considered such as gravity forces of the separate parts of body, resistance force of environment, shooting recoil force, rifle revolution moment and the Coriolis force. Total of muscles forces (muscles tractive forces) depends on the whole mechanical and anatomical characteristics of human body.

2.4 Structure of human body

From a biomechanical point of view, the support-locomotor apparatus of human is controlled by a biosystem consisting of chains, links and their joints with a group of muscles. A number of movable chains, movement degrees of freedom, nomenclature muscle groups and their interactions vary with the current human body position. The human skeleton represents a complex spatial construction from different kinds of nearly 207 bones and up to 400 muscles. The complexity of the human body structure and rifle necessitates using the development mathematical model and finite element approach. As can be observed in Fig. 2, segments of the human body are considered mass-inertia characteristics of the major muscles in FEM. The terms of each muscles are represented in the caption of Fig. 2. Their coordinates and center of mass are represented in Fig. 3.

Moment of human-rifle system is described by the following equation.

$$[M]\{\dot{\Delta}_{t+\delta t}\} + [H]\{\dot{\Delta}_{t+\delta t}\} + [K]\{\Delta_{t+\delta t}\} = \{P_{t+\delta t}\} \quad (5)$$

where $[H]$ is dissipation matrix in terms of experimental coefficients α and β as $[H] = \alpha[K] + \beta[M]$. If experimental data is absent, $[H] =$

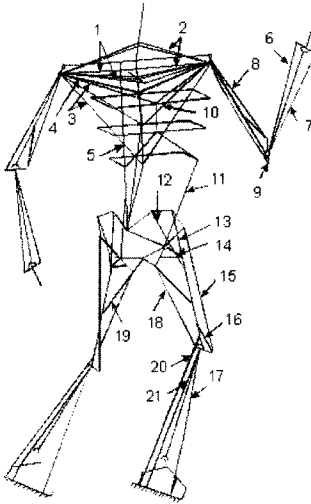
$0.15 \cdot 4\pi\nu[M]$ approximately according to Arsenev et al., 1996, where ν is the lowest natural fre-

quency of the system. $\{P_{t+\delta t}\}$ is a generalized exterior force vector with arbitrary law of change.

3. Human-Rifle System Model

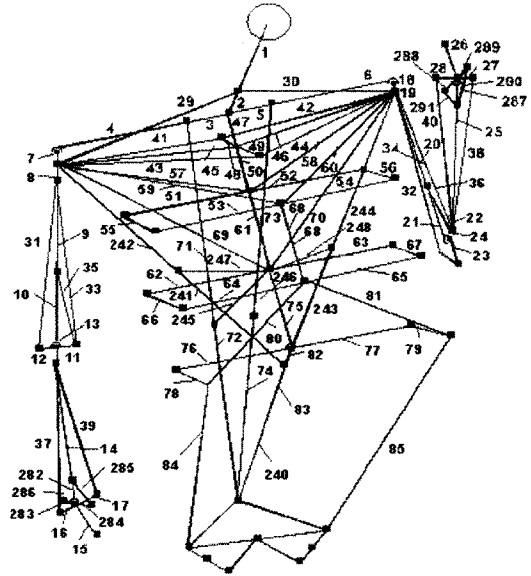
3.1 Finite element model

The human-rifle system is represented by finite elements and the system of equations (1) ~ (5) is

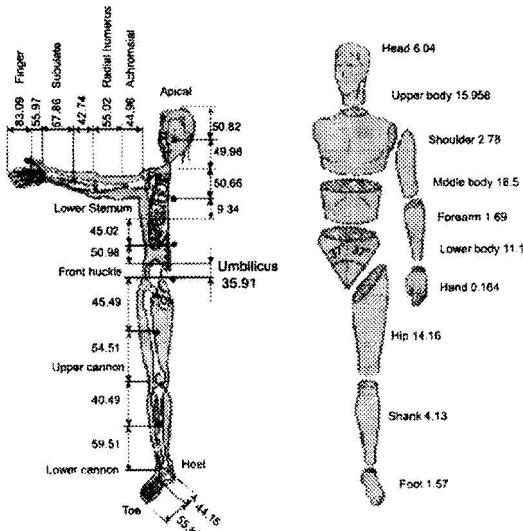


1. Supraspinal, subspinal and subscapular; 2. Trapeziform; 3. The most wide of back; 4. Big pectoral; 5. Long back; 6. Flexor of hand and fingers; 7. Extensor of hand and fingers; 8. Biceps; 9. Triceps; 10. Trapeziform+supraspinal, subspinal and subscapular; 11. Straight abdomen (stomach); 12. Big gluteus; 13. Middle and small gluteus; 14. Huckle-psoas; 15. Straight thigh; 16. Wide thigh; 17. Front tibial; 18. Long head of thigh biceps; 19. Long stretching+crested; 20. Plaice-like; 21. Head of sural

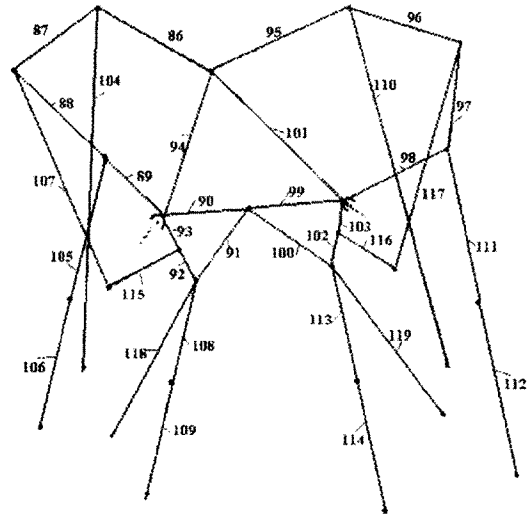
Fig. 2 The major muscles of human body segments considering anatomical characteristics



(a) Finite element model of the trunk

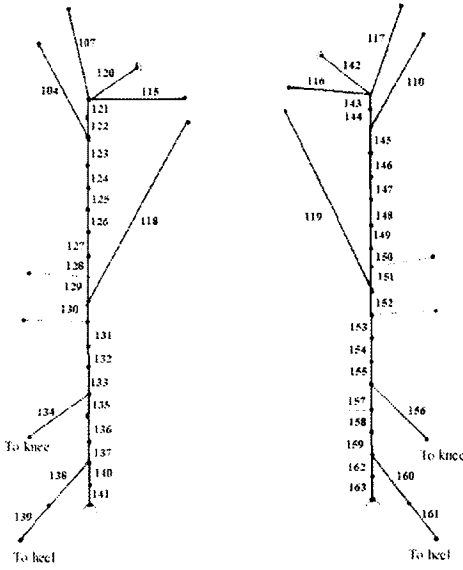


(a) Percentage of length (b) Percentage of mass
Fig. 3 Anthropometrical percentage of segment boundaries and center of mass

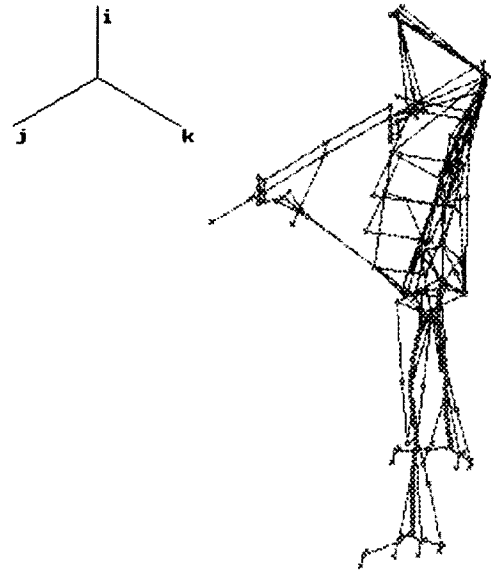


(b) Finite element model of the pelvis

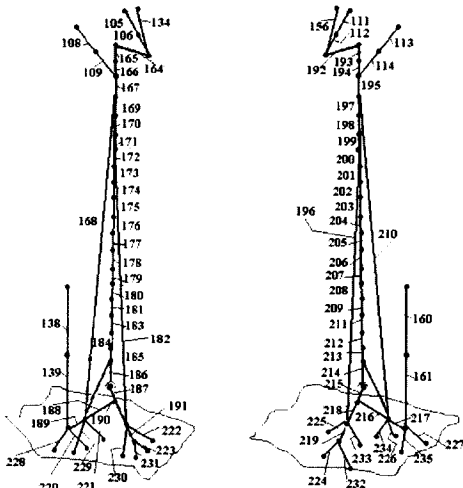
Fig. 4 Finite element model of the human body and rifle



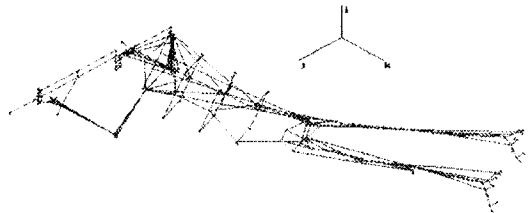
(c) Finite element model of the thigh



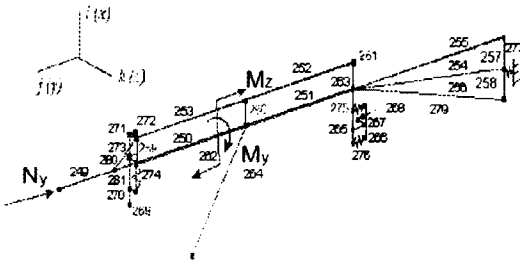
(a) Finite element model human-rifle system of the standing position



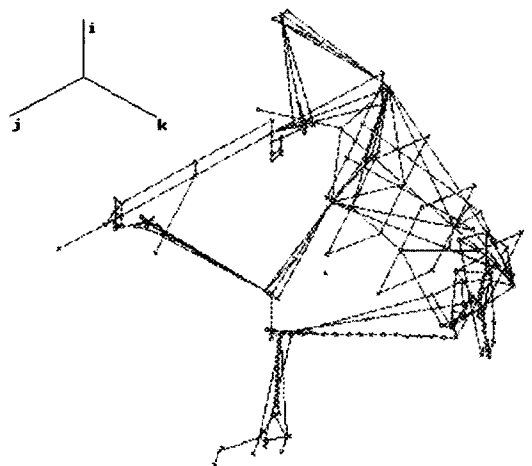
(d) Finite element model of the shank



(b) Finite element model human-rifle system of the prone position



(e) Finite element model of the rifle, 7.62 mm caliber small arms



(c) Finite element model human-rifle system of the kneeling position

Fig. 4 Finite element model of the human body and rifle

Fig. 5 Finite element model of human-rifle system in major positions

solved by the Newmark method. Also the generalized displacements and corresponding generalized velocities and accelerations of elements are determined.

The finite element models of the parts of the human body are shown in Figs. 4(a), 4(b), 4(c), 4(d), and besides finite element model of the 5.56 mm caliber rifle is shown in Fig. 4(e).

Finite element representations of human-rifle system for standing, kneeling and prone firing positions are shown in Fig. 5. From the biomechanical point of view, the standing position is the most complex unstable case of rifle-man. When standing, the center of gravity of the head must be maximal height by method of sensitivity analysis with descending gradient search method, and the stiffness characteristics of the muscles is determined.

3.2 Exterior forces

During shooting, the following forces act on the elements :

i) Longitudinal force $N_y(t)$ determined by change in pressure on barrel bore base P_{br} in formula $N_y(t) = 0.25(dx)^2 P_{br}$, where d is caliber, x is the degree of barrel chamber and shoulder ;

ii) Torque due to thread inside bore $M_y(t)$ determined by $M_y(t) = 0.5P_{br} \cdot dtga$, where a is bore groove angle ;

iii) Bending moment $M_z(t)$ under force $N_y(t)$ by $M_z(t) = P_{br} \cdot e$. The moment acts on axis z passing through the armament center of gravity and displaced bore axis on value e (arm of dynamical pare) ;

iv) The Coriolis force $F_k(t)$ is due to transportation motion during bullet or breach movement. This force is calculated on dependence $F_k(t) = 2q(V \cdot W)$, where q is mass of bullet or breach-block, where V is vector of bullet velocity, W is vector of angular velocity, the transportation velocity of barrel determining the results of dynamical solutions human-rifle system. The Coriolis force direction along axes x and y can change in opposite directions.

Forces and moments N_y , M_y and M_z are represented in Fig. 4(e) and calculated by program

INBAL. In order to estimate their minimal and maximal values, the calculations on their extreme deviations were carried out. During 0.005 second, the variation of bore base force $N_y(t)$ is given in Fig. 6 as function of time. In addition, calculated minimal, average and maximal values are represented in caption of Fig. 6. The variation of torque $M_z(t)$ is given in Fig. 7 and the variation of bending moment $M_y(t)$ is given in Fig. 8 as

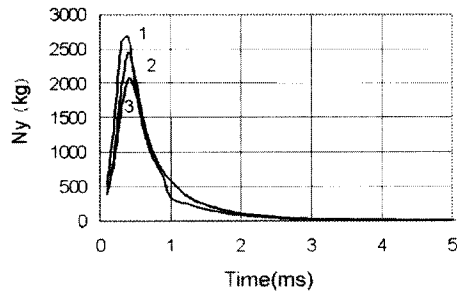


Fig. 6 Variation of bore base force : 1-maximal value ; 2-average value ; 3-minimal value

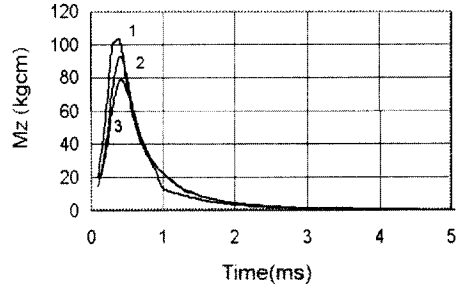


Fig. 7 Variation of torque : 1-maximal value ; 2-average value ; 3-minimal value

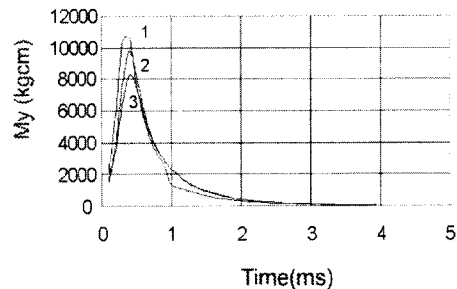


Fig.8 Variation of bending moment ($e=4$ cm) : 1-maximal value ; 2-average value ; 3-minimal value

function of time, where arm of dynamical pare e is equal to 4 cm. During 0.012 second, the variation of bullet velocity V is given in Fig. 9 as function of time.

More detailed Monte Carlo calculations allowed installation of the distribution laws very close to normal (Figs. 10 and 11).

In human biomechanics, the following para-

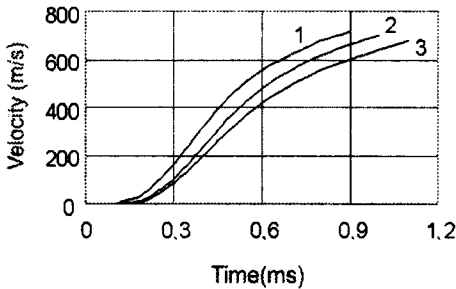


Fig. 9 Variation of bullet velocity: 1-maximal value; 2-average value; 3-minimal value

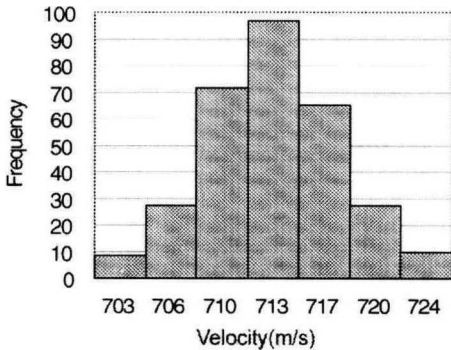


Fig. 10 Distribution of bullet muzzle velocity

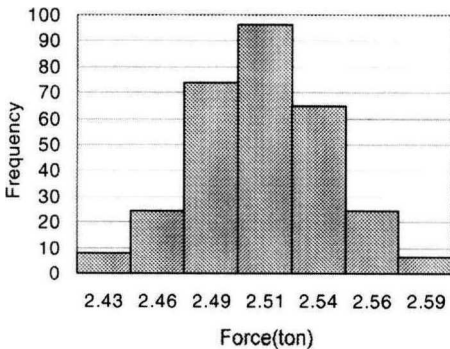


Fig. 11 Distribution of maximal bore base force

meters are randomly varied ; skeleton and muscles segments lengths, their cross-section areas, physical characteristics of bone-muscle formations (modulus of longitudinal elasticity and rigidity, densities, inertia moments). The arm of dynamical pare e is randomly varied also. Random variation of rifle parameters, ballistic and human parameters leads to linear and angular displacements of the muzzle that are complex function of time. In order to find characteristics of random deviations in the same time, the probability calculations are carried out. Firing 15 shots, Calculated results of vertical and horizontal angular displacements during 0.1 second are shown in Figs. 12 and 13.

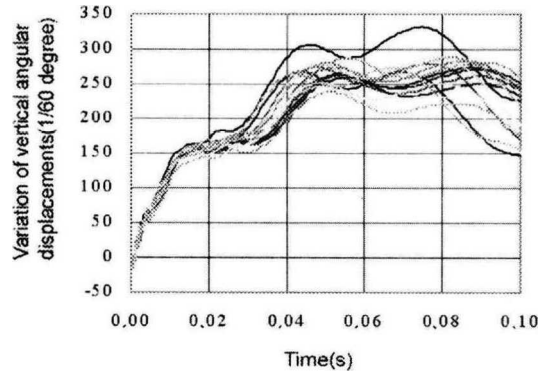


Fig. 12 Variation of random vertical angular displacements of barrel muzzle face (15 shots, 0.1 s)

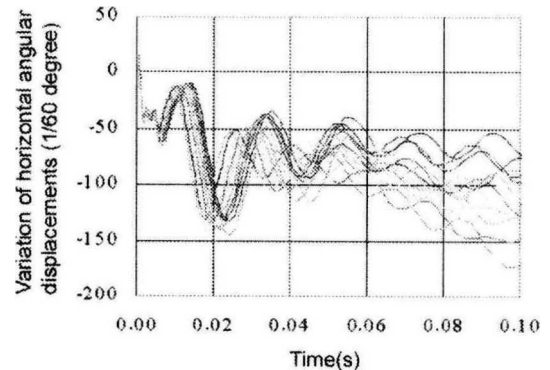


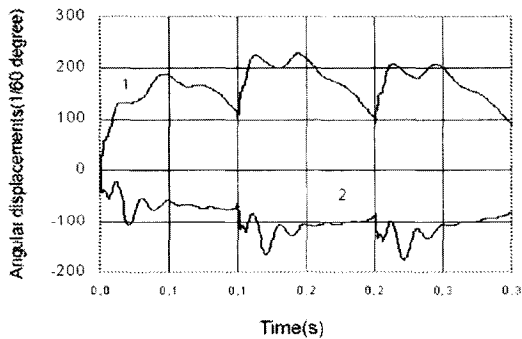
Fig. 13 Variation of random horizontal angular displacements of barrel muzzle face (15 shots, 0.1 s)

3.3 Exterior ballistics (EXBAL)

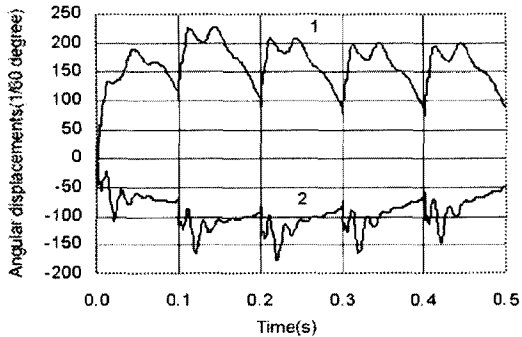
The mathematical model of exterior ballistics (Pravdin et al., 1999) includes the following assumptions :

- i) Bullet is a lengthy symmetrical body about the longitudinal axis ;
- ii) Target is immovable ;

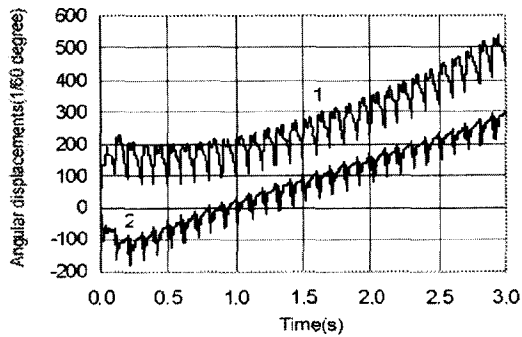
- iii) Acceleration of gravity is equal to 9.81 m/s^2 ;
- iv) Change of gravity acceleration depending on firing distance is neglected ;
- v) Stabilization of bullet is provided by its rotation ;
- vi) Real atmosphere is considered ;
- vii) Beginning of trajectory coincides with the departure point of bullet ;



(a) Burst length 3 shots

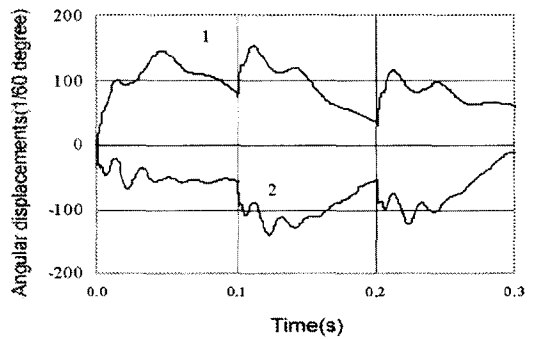


(b) Burst length 5 shots

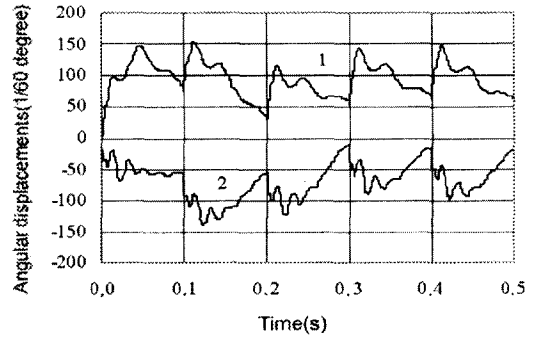


(c) Burst length 30 shots

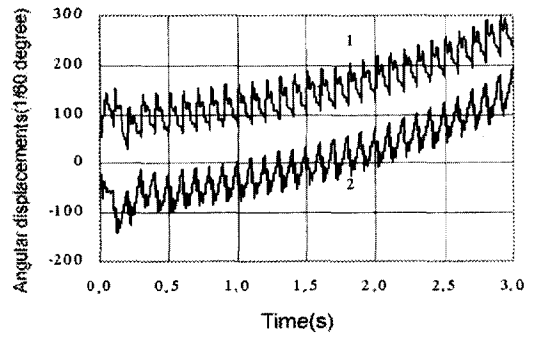
Fig. 14 Variation of the angular displacements of barrel muzzle face of the standing position : 1-vertical ; 2-horizontal



(a) Burst length 3 shots



(b) Burst length 5 shots



(c) Burst length 30 shots

Fig. 15 Variation of the angular displacements of barrel muzzle face of the kneeling position : 1-vertical ; 2-horizontal

- viii) The effects of the Earth rotation is neglected because of short firing range of small arms ;
- ix) The change of air temperature is neglected, because height of bullet flight is small ;
- x) Exterior ballistic model includes 47 equations and is solved by the fourth order Runge-Kutta method.

Change of the angular displacements of barrel muzzle face for bursts with 3, 5 and 30 shots with

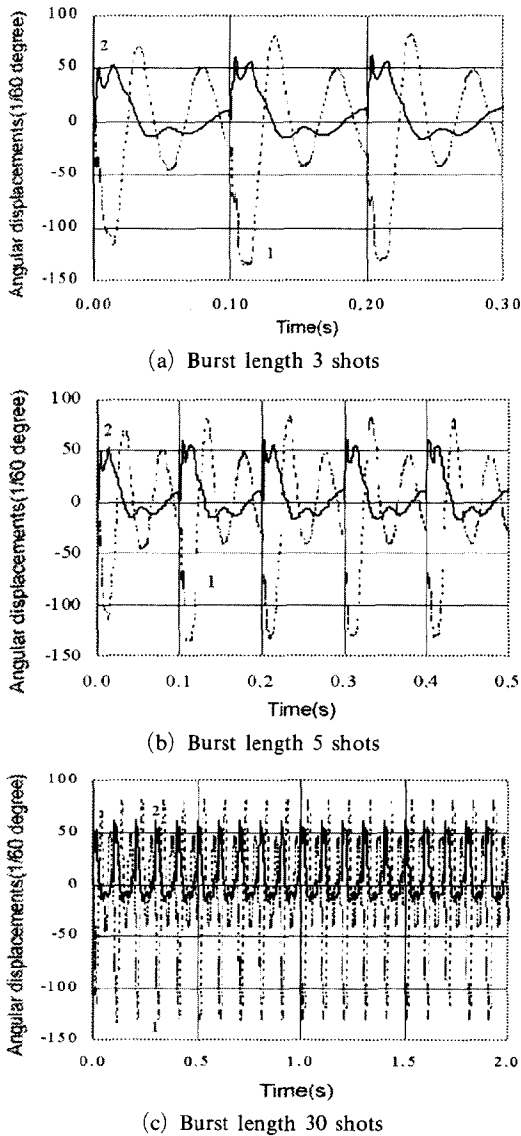


Fig. 16 Variation of changing the angular displacements of barrel muzzle face of the prone position : 1-vertical ; 2-horizontal

firing range 1,000 m is shown in Figs. 14 to 16. Figure 14 shows the variation of the angular displacements of barrel muzzle face of the standing position. Fig. 15 shows the results of the kneeling position and Fig. 16 shows the results of the prone position.

Figure 17 shows a large difference between dispersions of the first impact bullet points and the following ones for 4 bursts with 5 shots. Similar results can be computed for other firing ranges, 100, 500 and 600 m. Firing results are based on the calculation of dispersion impact points on the target.

Figure 18 shows the results of combined dispersion of 300 bullets for shooting from the stan-

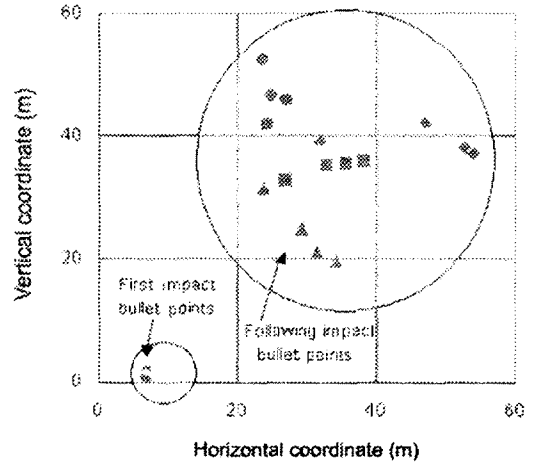


Fig. 17 Characteristics of random impact points coordinates of the standing position on distance 1,000 m (4 bursts with 5 shots)

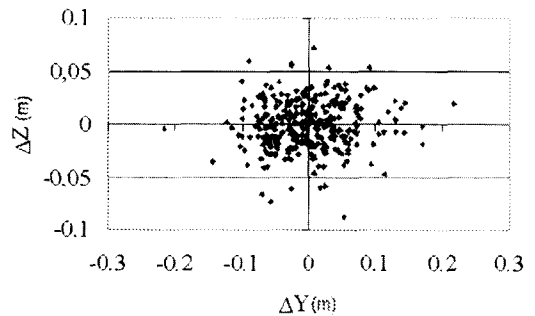


Fig. 18 Dispersion of impact points of the standing position without influence of human-rifle system for 300 shots

ding position of firing range 100 m by single fire without taking into account human-rifle system interaction. As the results of analysis, average dispersion of vertical direction D_{ver} is 0.0154 m at 100 m range, and average dispersion of horizontal direction D_{hor} is 0.039 m at same the range.

On the other hand, Fig. 19 shows the results of 300 bullets with taking into account human-rifle system interaction. The interaction of human-rifle system has dominant influence on dispersion parameters of bullets in the kind of environment, powder, bullet configuration and ballistics parameters. As the results of interaction analysis, average dispersion of vertical direction D_{ver} is 0.285 m at 100 m range, and average dispersion of horizontal direction D_{hor} is 0.255 m at the same range.

Table 1 shows influence of human-rifle interaction. Additionally it shows similarity between standard data of the firing test of real rifle and results of this human-rifle system interaction an-

Table 1 Comparison dispersion of the firing test of real rifle with results of this human-rifle system at 100 m range

Disp.	Analysis w/ interaction	Analysis w/o interaction	Firing test of real rifle
D_{ver} (m)	0.0154	0.285	0.288
D_{hor} (m)	0.039	0.255	0.258

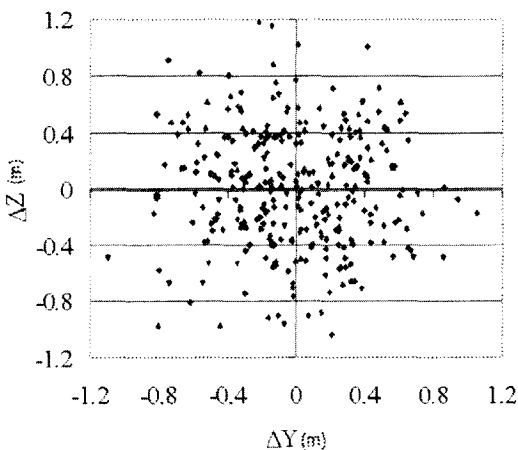


Fig. 19 Dispersion of impact points of the standing position with influence of human-rifle system for 300 shots

alysis. The difference between dispersions of standing position is only 3 mm at 100 m range in vertical and horizontal direction. Such error is kept within experiment processing errors and has high accuracy for statistical investigation

4. Conclusions

The description of a complex mathematical model of biomechanical interaction for human-rifle system is represented. This model can determine the calculations for three typical human positions, i.e., standing, kneeling and prone. The model includes the solution to interior/exterior ballistic processes and behavior of biomechanical human-rifle system. The numerical investigations are given in terms of influence of environment, bullet, powder, barrel geometry parameters, human and rifle anthropological and geometry parameters in the case of 7.62 mm small caliber machine gun.

The result of this study shows dispersion of the real firing and dispersion of human-rifle system analysis. The difference between dispersions is only 3 mm at 100 m range in vertical and horizontal directions. Each error percentage is 1.16% in vertical direction, and 1.04% in horizontal direction. It can be validated that human-rifle system has high accuracy for real firing dispersion as well as influence of human-rifle interaction.

This study makes it possible to investigate the influenced wide class of parameters, interior/exterior ballistics, rifle and human for assessment firing accuracy from different small caliber weapon system. Additionally this makes it possible to correct fire of personal riflemen taking into account their personal anthropological characteristics of different firing positions. Given model can be found without and with taking into account human-rifle system interaction, and can be applied to anthropology, biomechanics, medical science, gait analysis, interior ballistics and exterior ballistics.

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